MULTI-DOMAIN TOPOLOGY OPTIMIZATION USING ADVANCED SUBSTRUCTURING TECHNIQUE

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ABSTRACT

This paper summarizes a new capability in the topology optimization, which employs an advanced substructuring technique in the multi-domain topology optimization for the purpose of designing a complex and practical mechanical system. In the substructuring method, a structural system is divided into a series of substructures, which can be designed separately with the coupling of the subsystems. The condensation technique is applied to the substructures where the design changes are not considered at the current step. For the substructures where the optimal design is sought, the multi-domain topology optimization is applied in order to achieve the design objectives prescribed. The embedded advanced substructuring technique allows users to select nodal displacements and modal coordinates as generalized coordinates in the reduced-order models, and to use advanced Quasi-Static modes for high efficient modal analyses. Examples are illustrated in order to demonstrate the feasibility of the proposed technique to the topology optimization problem.

KEY WORDS: Topology Optimization, Substructuring Technique, Automotive Design

1. INTRODUCTION

Topology optimization (1-4) has been widely adopted in the design process for developing light-weight and high-performance structures, especially, in automotive vehicle structural systems. A bottleneck, however, is how to apply the topology optimization technique to deal with a large and complex engineering structure, such as a major subsystem in a full vehicle system. In the

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Report Documentation Page

1. REPORT DATE **01 JUL 2006**	2. REPORT TYPE **Journal Article**	3. DATES COVERED **04-06-2006 to 23-06-2006**

4. TITLE AND SUBTITLE **MULTI-DOMAIN TOPOLOGY OPTIMIZATION USING ADVANCED SUBSTRUCTURING TECHNIQUE**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) **Zheng-Dong Ma; Dongying Jiang; Noboru Kikuchi; Christophe Pierre; Basavaraju Raju**	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Department of Mechanical Engineering,University of Michigan,530 S State St,Ann Arbor,Mi,48109**	8. PERFORMING ORGANIZATION REPORT NUMBER **; #14003**

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, Mi, 48397-5000**	10. SPONSOR/MONITOR'S ACRONYM(S) **TARDEC**
	11. SPONSOR/MONITOR'S REPORT NUMBER(S) **#14003**

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
submitted to journal of applied mechanics

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15. SUBJECT TERMS
Topology Optimization, Substructuring Technique, Automotive Design

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Public Release**	**11**	

practical design, a structural system may contain a large number of components and substructures and each substructure may have a large number (hundreds of thousands) degrees of freedom in its discretized finite element model. The direct adoption of the whole system in a topology optimization process is inefficient and even formidable from the aspect of computational resource and the aspect to find a solution for the optimization problem.

This research aims at the implementation of advanced substructuring techniques (11,16-19) in the multi-domain topology optimization for designing complex mechanical systems. In the substructuring method (17-19), a mechanical system is divided into a series of, or multileveled, substructures, which can be designed separately with the coupling effects of the subsystems. The advanced substructuring technique allows users to select nodal displacements at the interfaces of the substructures and modal coordinate as generalized coordinates in the reduced-order model, and to use advanced Quasi-Static modes (11,17) for high efficient modal analyses. The condensation technique can be applied to the substructures where the design change is not considered at the current step. For the substructures where the optimal design is sought, the multi-domain topology optimization (20) can be applied for those subdomains in order to achieve the design objectives of the whole structural system. Another usage of this approach is that, the topology optimization can be zoomed into any subdomain within a design domain at anytime in order to lay out the details of the design at local area, or to reduce the stress concentration at a localized location. Using this divide-and-conquer design methodology, an optimal structural design can be obtained for the coupled substructures with reduced computational cost and better solvability of the design problem of large size, which are crucial in the application of the topology optimization technique for real engineering design problems.

2. MULTI-DOMAIN TOPOLOGY OPTIMIZATION

A multi-domain topology optimization (MDTO) approach is proposed in (20), which can be considered as a generalization of the standard, single-domain topology optimization developed in the earlier stage (5-9). In contrast to single-domain topology optimization, in which a given amount of the material is assigned to the entire design domain, MDTO allows the designer to assign different amounts of the material, or even different materials, to the various subdomains of the structure. Figure 1 depicts a structural domain that is divided into three subdomains, where a certain amount of material A is assigned to Subdomain 1, a different amount of material B is distributed into Subdomain 2, and Subdomain 3 is considered as a non-design domain, for which the material distribution is not allowed to change at the current design stage.
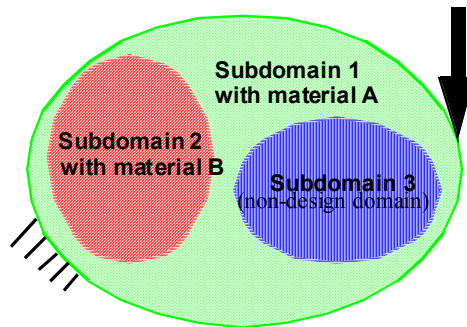

Figure 1. A multi-domain topology optimization problem

In the general case, the MDTO problem can be formulated as

$$
\begin{aligned}
&\text{Minimize} \ \ f(\mathbf{X}) \\
&\text{Subject to:} \ h_j(\mathbf{X}) \le 0 \quad (j = 1, 2, ..., m) \\
&\qquad\qquad \underline{x}_i \le x_i \le \overline{x}_i \quad (i = 1, 2, ..., n) \\
&\text{(and State Equations)}
\end{aligned}
\tag{1}
$$

where $f = f(\mathbf{X})$ denotes the objective function, $h_j = h_j(\mathbf{X})$ denotes the j-th constraint function for the volume (or weight) of the j-th substructure in the j-th subdomain (where $j = 1, 2, \cdots, m$); m is the total number of the subdomains; $\mathbf{X} = \{x_1, x_2, ..., x_n\}^T$ denotes the vector of the design variables, where n is the total number of the design variables; and $\underline{x}_i$ and $\overline{x}_i$ are the lower and upper bounds of design variable x_i, respectively. Note that $f(\mathbf{X})$ in Eq. (1) also needs to satisfy the state equations for the structural problem at hand. These state equations may include, for example, the static equilibrium equation, the equation that defines the free vibration eigenvalue problem, or the equation for the dynamic forced response.

The basic idea in the topology optimization technique proposed by Bendsøe and Kikuchi in 1988 (4) is to transform the optimal topology design problem into an equivalent Optimal Material Distribution (OMD) problem, as illustrated in Fig. 2. Here the structural domain is assumed to be filled with a non-homogeneous composite material characterized by a variable microstructure. A typical microstructure is formed inside an empty rectangle in a unit cell with three design variables a, b and θ, which are dimensions and orientation of the microstructure, respectively. Using homogenization, the effective material constants can be obtained as (4):

$$
\mathbf{D}^H = \mathbf{D}^H(a, b, \theta), \quad \rho^H = \rho^H(a, b)
\tag{2}
$$

where $\mathbf{D}^H$ denotes the effective elastic coefficient matrix at the material point, and ρ^H is the associated effective mass density. Both $\mathbf{D}^H$ and ρ^H are functions of the design variables a and b, and $\mathbf{D}^H$ is also a function of the orientation variable θ at the material point.

In the optimization process, the microstructure can vary anywhere between "empty" ($a=b=0$) and "solid" ($a=b=1$) using the design variables a and b, and it can be rotated using the orientation variable θ. Therefore, if one assumes that the total amount of the material, which is prescribed for the design problem at hand, remains constant in the optimization process, then the material will be moved from a region of the structural domain into another region to produce a new distribution of the material. Structural optimization technique (10,12-15) has been used to search for the optimal material distribution. By moving and orienting the material so as to improve the objective function of the optimization problem, one can finally obtain an OMD that corresponds to the optimal structure. Various microstructure models have been developed based on this fundamental concept, including artificial material models, which represent a simplification of this concept (1-4).
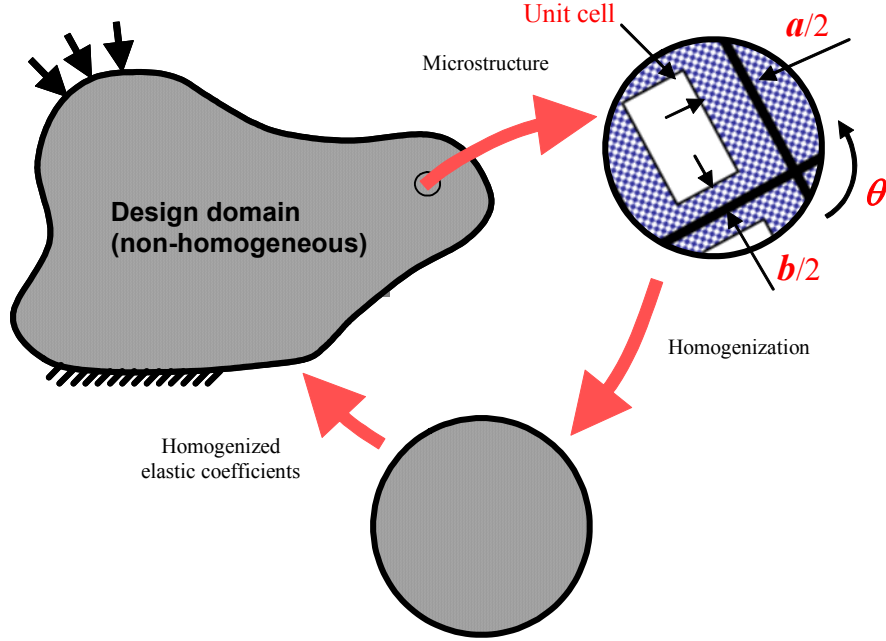
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Figure 2. Basic concept of the homogenization-based topology optimization method

3. ADVANCED SUBSTRUCTURING METHODS

Using a standard finite element method, the state equation governing the dynamic response of the i-th structure shown in Fig. 1 can be obtained as:

$$\mathbf{M}^{(i)}\ddot{\mathbf{u}}^{(i)} + \mathbf{C}^{(i)}\dot{\mathbf{u}}^{(i)} + \mathbf{K}^{(i)}\mathbf{u}^{(i)} = \mathbf{f}^{(i)} \tag{3}$$

where $\mathbf{M}^{(i)}$, $\mathbf{C}^{(i)}$, and $\mathbf{K}^{(i)}$ are the mass, damping, and stiffness matrices of the i-th substructure in the system, respectively. $\mathbf{M}^{(i)}$ and $\mathbf{K}^{(i)}$ can be obtained by assembling the elementary mass and stiffness matrices as

$$\mathbf{M}^{(i)} = \mathop{A}_{e=1}^{n_{el}}{}^{(i)}\,\mathbf{m}_e \quad \text{and} \quad \mathbf{K}^{(i)} = \mathop{A}_{e=1}^{n_{el}}{}^{(i)}\,\mathbf{k}_e \tag{4}$$

where $\mathbf{m}_e = \mathbf{m}_e(a_e, b_e)$ and $\mathbf{k}_e = \mathbf{k}_e(a_e, b_e, \theta_e)$ are the mass and stiffness matrices of the finite element e, which are functions of a_e, b_e, and θ_e, where a_e, b_e, and θ_e are the discretized design variables of a, b, and θ, respectively, at the elementary level ($e = 1, 2, \cdots, n_{el}$). In the special case of proportional damping, the viscous damping matrix $\mathbf{C}^{(i)}$ in Eq. (3) can be obtained as $\mathbf{C}^{(i)} = \beta_i \mathbf{K}^{(i)}$, where β_i is the damping coefficient of the i-th substructure.

State equations for the topology optimization problem, Eq. (3), usually feature a very large number of variables (typically the nodal displacements). Even for a simple structure, a finely discretized finite element model can involve from thousands to millions of nodal variables. In topology optimization, such fine meshes are usually required for obtaining smooth boundaries and interfaces. Substructuring methods provide an effective tool for condensing the analysis

variables into a much smaller set, thus greatly reducing computer memory requirements and increase computational efficiency. This approach is particularly natural and useful for multi-domain topology optimization problems. While substructuring can be used in several ways to improve computational efficiency, in this paper the condensed analysis variables are those associated with the non-design subdomains of the structure (see Fig. 1). The nodal displacement variables can be condensed into those at the boundaries of the subdomain when a static analysis is considered, and with an additional small number of generalized modal coordinates when a dynamic problem is considered. Note that the design and non-design subdomains can be switched if necessary during the different design stages.

In the case of a static condensation, assuming that $\mathbf{u}_a$ denotes the vector of nodal displacements associated with the active nodes (usually the boundary nodes) of a non-design subdomain, and $\mathbf{u}_o$ denotes the vector of the nodal displacements associated with the other nodes (usually the internal nodes) of the same subdomain, the state equation for the subdomain can be written as

$$\begin{bmatrix} \mathbf{k}_{oo} & \mathbf{k}_{oa} \\ \mathbf{k}_{ao} & \mathbf{k}_{aa} \end{bmatrix}^{(i)} \begin{Bmatrix} \mathbf{u}_o \\ \mathbf{u}_a \end{Bmatrix}^{(i)} = \begin{Bmatrix} \mathbf{f}_o \\ \mathbf{f}_a \end{Bmatrix}^{(i)} \tag{5}$$

where $\mathbf{k}_{ij} (i, j = o, a)$ are the blocks of the stiffness matrix of the subdomain associated with $\mathbf{u}_o$ and $\mathbf{u}_a$, respectively, and $\mathbf{f}_o$ and $\mathbf{f}_a$ are the corresponding nodal force vectors. Then Eq. (5) can be condensed into an equation that involves only $\mathbf{u}_a$, namely,

$$\mathbf{k}_{aa}^* \mathbf{u}_a = \mathbf{f}_a^* \tag{6}$$

where $\mathbf{k}_{aa}^* = \mathbf{k}_{aa} - \mathbf{k}_{ao} \mathbf{k}_{oo}^{-1} \mathbf{k}_{oa}$ and $\mathbf{f}_a^* = \mathbf{f}_a - \mathbf{k}_{ao} \mathbf{k}_{oo}^{-1} \mathbf{f}_o$. For the sake of simplicity, but not losing the generality, in the following discussions, we assume $\mathbf{f}_o = 0$, and therefore we have $\mathbf{f}_a^* = \mathbf{f}_a$.

Note that the coordinate transformation for the condensation is

$$\mathbf{u} = \begin{Bmatrix} \mathbf{u}_o \\ \mathbf{u}_a \end{Bmatrix} = \mathbf{C} \mathbf{u}_a \tag{7}$$

where the transformation matrix is

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{oa} \\ \mathbf{C}_{aa} \end{bmatrix} = \begin{bmatrix} -\mathbf{k}_{oo}^{-1} \mathbf{k}_{oa} \\ \mathbf{I} \end{bmatrix} \tag{8}$$

Each column of $\mathbf{C}$ is called a "*static mode*" or a "*constraint mode*".

In the case of a dynamic condensation, the state equation can be written as

$$\begin{bmatrix} \mathbf{k}_{oo} & \mathbf{k}_{oa} \\ \mathbf{k}_{ao} & \mathbf{k}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_o \\ \mathbf{u}_a \end{Bmatrix} + \begin{bmatrix} \mathbf{m}_{oo} & \mathbf{m}_{oa} \\ \mathbf{m}_{ao} & \mathbf{m}_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_o \\ \ddot{\mathbf{u}}_a \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_a \end{Bmatrix} \tag{9}$$

where $\mathbf{m}_{ij}\,(i,j=o,a)$ denote the blocks of the mass matrix. We now divide the active nodal set (*a*-set) into two subsets *p*-set and *s*-set (namely $a=p\otimes s$), where the *s*-set is defined by those nodes that will be fixed in the eigenmodes extraction process, and the *p*-set is defined by the other nodes (those will be free in the eigenmodes extraction process). Defining an eigenvalue problem with respect to the nodal set $n=o\otimes p$, namely

$$\left(\begin{bmatrix}\mathbf{k}_{oo} & \mathbf{k}_{op}\\\mathbf{k}_{po} & \mathbf{k}_{pp}\end{bmatrix}-\lambda_n\begin{bmatrix}\mathbf{m}_{oo} & \mathbf{m}_{op}\\\mathbf{m}_{po} & \mathbf{m}_{pp}\end{bmatrix}\right)\left\{\begin{matrix}\boldsymbol{\varphi}_{on}\\\boldsymbol{\varphi}_{pn}\end{matrix}\right\}=0 \tag{10}$$

we now can obtain a set of normal modes from Eq. (10), namely $\boldsymbol{\Phi}=\begin{bmatrix}\boldsymbol{\Phi}_{on}\\\boldsymbol{\Phi}_{pn}\end{bmatrix}$.

Using the normal modes calculated from Eq. (10), the transformation equation, Eq. (8) is extended as

$$\left\{\begin{matrix}\mathbf{u}_o\\\mathbf{u}_a\end{matrix}\right\}=\mathbf{D}\left\{\begin{matrix}\mathbf{q}_n\\\mathbf{u}_a\end{matrix}\right\} \tag{11}$$

where $\mathbf{q}_n$ is a vector of so-called generalized modal coordinates. The transformation matrix $\mathbf{D}$ for the dynamic problem can then be obtained as

$$\mathbf{D}=\begin{bmatrix}\boldsymbol{\Phi}_{on}-\boldsymbol{\Psi}_{op}\boldsymbol{\Phi}_{pn} & \boldsymbol{\Psi}_{oa}\\0 & \mathbf{I}\end{bmatrix} \tag{11}$$

The matrix $\boldsymbol{\Psi}=\begin{bmatrix}\boldsymbol{\Psi}_{oa}\\\mathbf{I}\end{bmatrix}$ contains the so-called "*quasi-static modes*" (QSM) (11,17), where $\boldsymbol{\Psi}_{oa}=\begin{bmatrix}\boldsymbol{\Psi}_{op} & \boldsymbol{\Psi}_{os}\end{bmatrix}$, which can be calculated by solving the quasi-static (frequency response) problem associated with Eq. (2). Theoretically, $\boldsymbol{\Psi}$ can be written as

$$\boldsymbol{\Psi}=\begin{bmatrix}-\left(\mathbf{k}_{oo}-\omega_c^2\mathbf{m}_{oo}\right)^{-1}\left(\mathbf{k}_{oa}-\omega_c^2\mathbf{m}_{oa}\right)\\\mathbf{I}\end{bmatrix} \tag{12}$$

where ω_c is the "*central frequency*," which is determined by the frequency range of interest (11,17). Note that the size of $\mathbf{q}_n$ in Eq. (10) is usually far smaller than $\mathbf{u}_o$, therefore, the coordinate transformation Eq. (11) (as well as Eq. (8)) significantly reduces the size of the analysis problem. For a static analysis, Eq. (8) does not induce any error into the original analysis problem. But for a dynamic analysis problem, Eq. (11) is an approximation, and the error can be controlled by properly choosing the central frequency and/or considering additional modal coordinates in Eq. (11). Note that for $\omega_c=0$, the QSM defined in Eq. (12) reduce to the traditional *static (constraint) modes* proposed by Hurty (18) and Craig and Bampton (19), which are given in Eq. (8). Compared with the static modes, the QSM can significantly reduce both the size of the analysis problem and the error induced by the coordinate reduction process,

particularly within the frequency range of interest. This is because not only the higher-frequency modes are truncated, but also the lower-frequency modes outside the frequency range of interest. Therefore, the QSM can handle dynamic response problems within higher frequency ranges, and they include the traditional static modes as a special case. Note that the central frequency is typically selected at the middle of the frequency domain of interest, and that multiple central frequencies can also be used (17). Finally, note that if *p*-set defined in Eq. (10) is null, then Eq. (11) is reduced to a fixed-interface CMS method and the Craig-Bampton method can be considered as a special case of it when ω_c=0. On the other hand, if *p*-set becomes *a*-set, the Eq. (11) is reduced to a so-called free-interface CMS method.

4. EXAMPLES

4.1 Refine design in a zoomed-in area A cantilevered sandwich beam design problem is shown in Fig. 3 to illustrate the multi-domain topology optimization along with the substructuring technique. The external force, **F**, represents a combination load of pushing in axial direction and bending in lateral direction. Four design subdomains are specified in Fig. 3, and they are shown in green and yellow. The blue non-design domains are filled with solid-cell material (*a*=*b*=1 in Eq. 2). The design objective is to obtain an optimal material distribution in the design domain so that the cantilevered beam has the maximum rigidity under the specified external load, **F**.
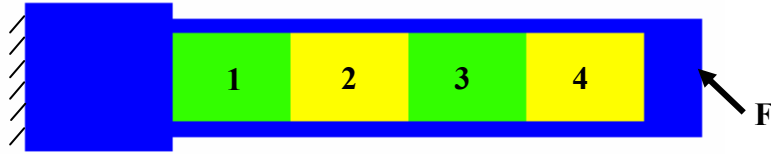

Figure 3. A cantilevered sandwich beam design problem.

A multi-domain topology optimization problem is considered with a finite element model of 3,906 degrees-of-freedom. The material density for each design subdomain is given as 20% with an even distribution among the subdomains.

A converged optimal material distribution for the first design problem is shown in Fig. 4-a. The von Mises stress resulted from this design is shown in Fig. 4-a, with the normalized maximum stress 16.6, occurring at the elements near subdomain 1 and 2. In order to reduce the maximum stress in the first design, additional 13% material is added to subdomains 1 and 2 while other design domains are fixed at the current step. The substructuring technique is utilized to condense the DOFs in subdomains 3 and 4, which reduce the total DOFs from 3,906 to 960. The design domain in the second step optimization is shown in the grey shadow in Fig. 4-b.

After the second step topology optimization, the maximum von Mises stress is then reduced to 13.7 as shown in Fig. 4-c. To further reduce the maximum stress in the current design, the topology optimization is zoomed into two new small design domains near the stress concentration areas as shown in Fig. 4-d in grey color. All the remaining subdomains, the original design subdomains 1-4 and the blue non-design domain in Fig. 3, are condensed through the substructuring technique. The total DOFs of the FE model for the final stage optimal design

becomes 336, which is significantly smaller than the original 3,906 in the original optimization problem. The final optimal design is shown in Fig. 4-e. The maximum von Mises stress is now reduced to 10.4 from the maximum stress 16.6 in the first step optimization by adding a small percentage of the material.
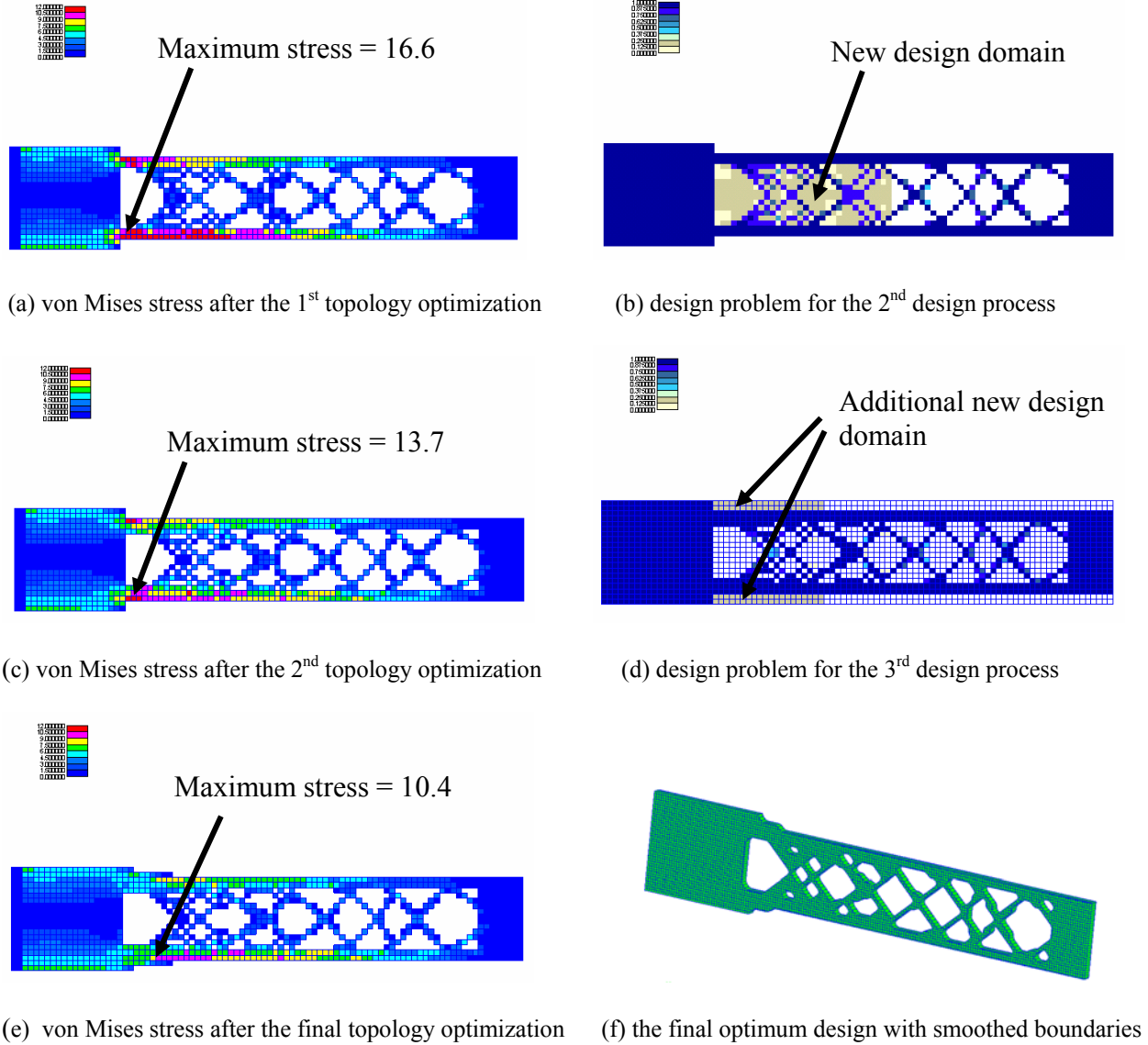

(a) von Mises stress after the 1st topology optimization

(b) design problem for the 2nd design process

(c) von Mises stress after the 2nd topology optimization

(d) design problem for the 3rd design process

(e) von Mises stress after the final topology optimization

(f) the final optimum design with smoothed boundaries

Figure 4. The optimization design procedure and the results

Through the three-stage multi-domain topology optimization, an optimal design has been obtained with the maximum rigidity and significantly reduced maximum stress, the later improves the strength of the structure. The maximum stress is reduced with the substructuring-technique-based refine design procedure developed in this paper.

4.2 Coupled substructures design The second example demonstrates a follow-up optimal design problem for the cantilevered beam illustrated in section 4.1. Given the optimal design layout shown in Fig. 4-f, we now consider an optimal fixture design problem for the structure.

The left-end fixture part is redesigned using the topology optimization process with reduced amount of material for the fixture. The initial design problem is shown in Fig. 5-a, while light blue color indicates the design domain for the fixture.

In Fig. 5-a, the material constraint for fixture design domain is 37.5%. The optimal structure obtained in section 4.1 is condensed with substructuring technique. The final DOFs of the FE model used in current optimization is 914, while the DOFs of the full FE model is 3,906. The design result for the fixture is shown in Fig. 5-b,c,d. It is seen that materials in the design domain are moved to the upper and lower edges to withstand the lateral bending load and a hole is opened in the middle in order to reduce the weight of the fixture. The final optimal design is shown in Fig. 5-d with the smoothed boundaries.
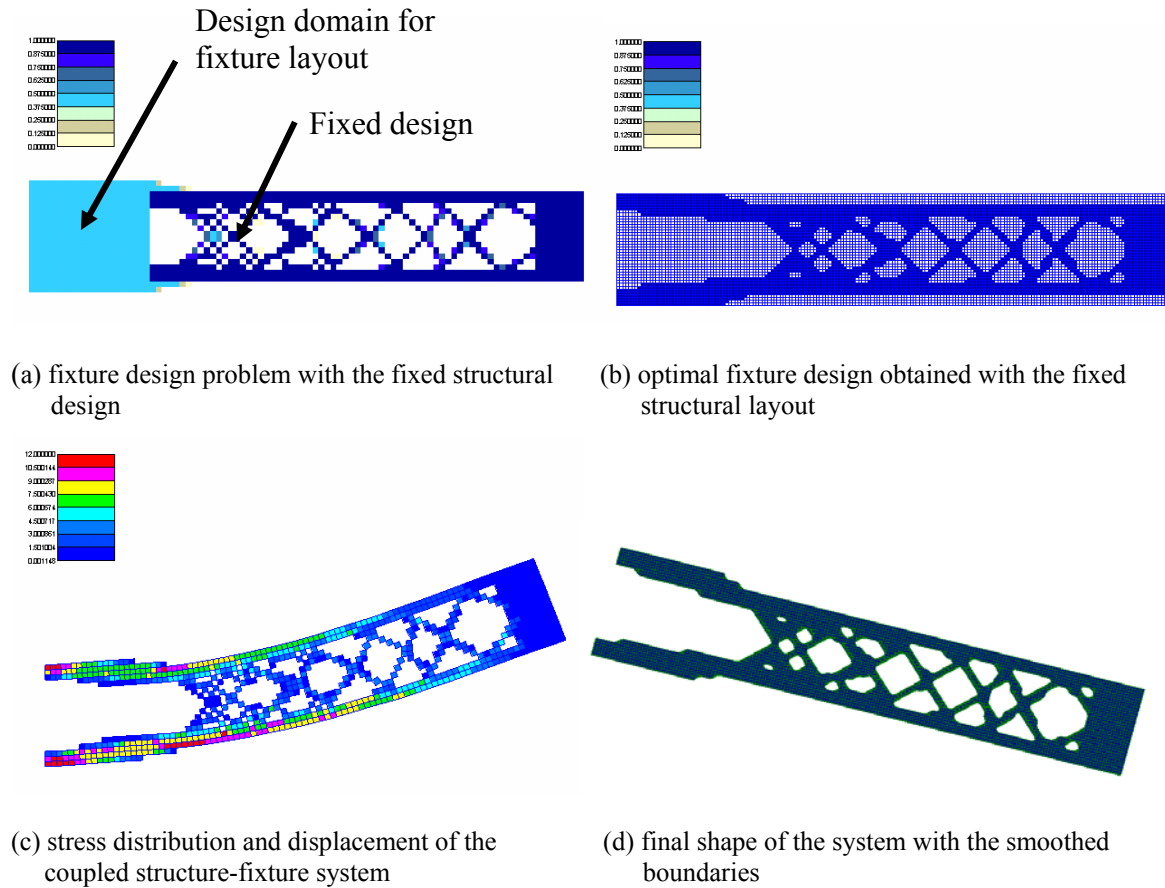
Design domain for fixture layout

Fixed design

(a) fixture design problem with the fixed structural design

(b) optimal fixture design obtained with the fixed structural layout

(c) stress distribution and displacement of the coupled structure-fixture system

(d) final shape of the system with the smoothed boundaries

Figure 5. The topology optimization in the fixture region.

5. CONCLUSIONS

An optimal design problem is illustrated to demonstrate the benefits from embedding the substructuring technique in the multi-domain topology optimization process. It is seen that the substructuring technique can reduce the size of the finite element model, thus improves the solvability of the design problem. Stress concentration can be overcome efficiently using the

zoomed-in topology optimization in the neighborhood of the stress concentration. The proposed substructuring technique can be also applied to dynamic problems such as the eigenmode optimization, and the frequency response optimization, and be readily extended to deal with three-dimension design problems for complex mechanical systems.

6. ACKNOWLEDGEMENT

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